Finite Math - J-term 2019
Lecture Notes - 1/24/2019

Homework

- Section 7.2 - 7, 8, 9, 10, 15, 16, 17, 18, 27, 28, 30, 31, 33, 34, 35, 36, 40
- Section 7.3 - 7, 8, 9, 10, 13, 17, 19, 35, 36, 52

Section 7.2 - Sets

Set Properties and Notation. A *set* is a collection of objects specified in such a way that we can tell whether any given object is in the set. We usually will denote sets by capital letters. Objects in a set are called *elements* or *members* of the set. Symbolically,

$a \in A$ means "a is an element of the set A"
$a \notin A$ means "a is not an element of the set A"

It is possible to have a set without any elements in it. We call this set the *empty set* or *null set*. We denote this set by $\varnothing$. An example of a set which is empty is the set of all people who have been to Mars.

We often denote sets by listing their elements between a pair of braces: { }. For example, the following are sets:

$$\{0, 1, 2, 3, 4, 5\}, \{a, b, c, d, e\}, \{1, 2, 3, 4, 5, ...\}.$$

Another common way to write sets is by writing a rule in between braces. For example,

$$\{x|x \text{ is even}\}, \{x|x \text{ has hit more than 50 home runs in a single season}\}, \{z|z^2 = 1\}.$$

The way to read this second type of set is, for example, "the set of x such that x is even" or "the set of z such that $z^2 = 1$. Notice that we get two kinds of sets like this: *finite sets* (the set only has finitely many elements) and *infinite sets* (the set has infinitely many elements). The sets $\{1, 2, 3, 4, 5...\}$ and $\{x|x \text{ is even}\}$ are infinite sets while the others are finite.

Example 1. *Let G be the set of all numbers whose square is* 9*.*

(a) *Denote G by writing a set with a rule (the second style above).*

(b) *Denote G by listing the elements (the first style above).*

(c) *Indicate whether the following are true or false: $3 \in G$, $9 \in G$, $-3 \notin G$.*

Solution.

Suppose we have two sets A and B. If every element in the set A is also in the set B, we say that A is a *subset* of B. By definition, every set is a subset of itself. If A and B have the exact same elements, then we say the sets are *equal*. Here is some notation for this:

$A \subset B$	means	"A is a subset of the set B"
$A \not\subset B$	means	"A is not a subset of the set B"
$A = B$	means	"A and B have the exact same elements"
$A \neq B$	means	"A and B do not have the exact same elements"

It follows that $\varnothing$ is a subset of every set and if $A \subset B$ and $B \subset A$, then $A = B$.

Example 2. *Let* $A = \{-3, -1, 1, 3\}$, $B = \{3, -3, 1, -1\}$, $C = \{-3, -2, -1, 0, 1, 2, 3\}$. *Decide the truth of the following statements*

$$\begin{array}{ccc} A = B & A \subset C & A \subset B \\ C \neq A & C \not\subset A & B \subset A \\ \varnothing \subset A & \varnothing \subset C & \varnothing \notin A \end{array}$$

Example 3. *Let* $A = \{0, 2, 4, 6\}$, $B = \{0, 1, 2, 3, 4, 5, 6\}$, $C = \{2, 6, 0, 4\}$. *Decide the truth of the following statements*

$$\begin{array}{ccc} A \subset B & A \subset C & A = C \\ C \subset B & B \not\subset A & \varnothing \subset B \\ 0 \in C & A \notin B & B \subset C \end{array}$$

Example 4. *Find all subsets of the following sets:*

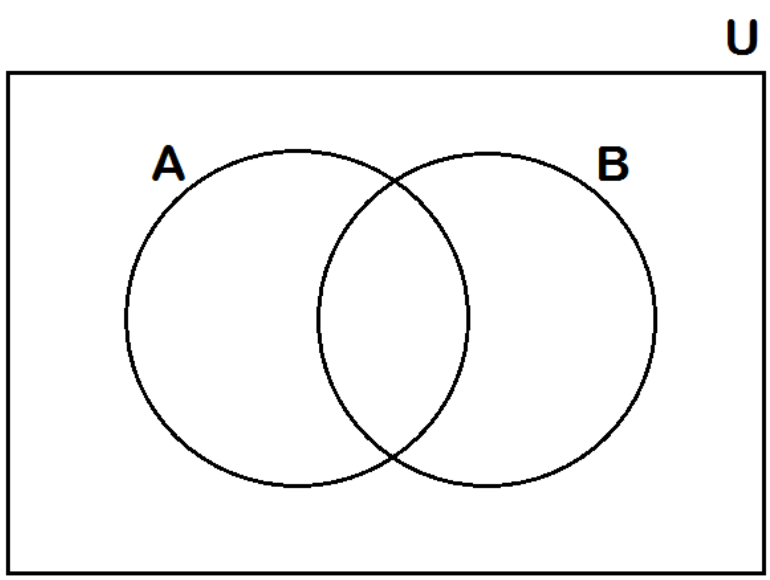
(a) $\{a, b\}$

(b) $\{1, 2, 3\}$

(c) $\{\alpha, \beta, \gamma, \delta\}$

Venn Diagrams and Set Operations. Given sets, there are various operations we can perform with them. To see these, it can be useful to visualize these with Venn Diagrams. First, we imagine that all of the sets in our problem live in some *universal set*, which we will denote by U, that is, we will assume that all of our sets are subsets of U.

To illustrate the set operations, we will use both actual sets and Venn diagrams. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 4, 5\}$, and $B = \{3, 4, 5, 6, 7\}$. For the Venn diagram, we will shade in the relevant regions of this diagram:



We have the following definitions:

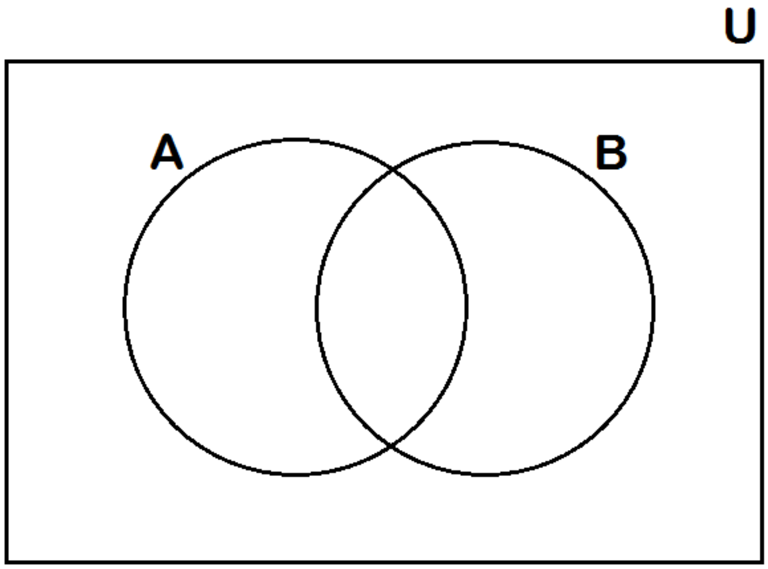
Definition 1 (Union). *The* union *of two sets A and B is the new set, denoted $A \cup B$, which consists of all elements which are in A or in B.*

$$A \cup B = \{x | x \in A \text{ or } x \in B\}.$$

Using the sets above,

$$A \cup B =$$

and as a Venn Diagram



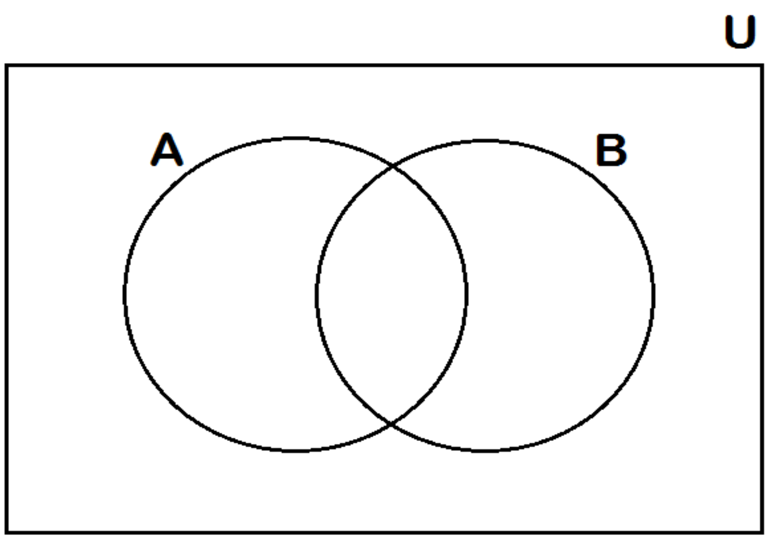
Definition 2 (Intersection). *The* intersection *of two sets A and B is the new set, denoted $A \cap B$, which consists of all elements which are in A and in B.*

$$A \cap B = \{x | x \in A \text{ and } x \in B\}.$$

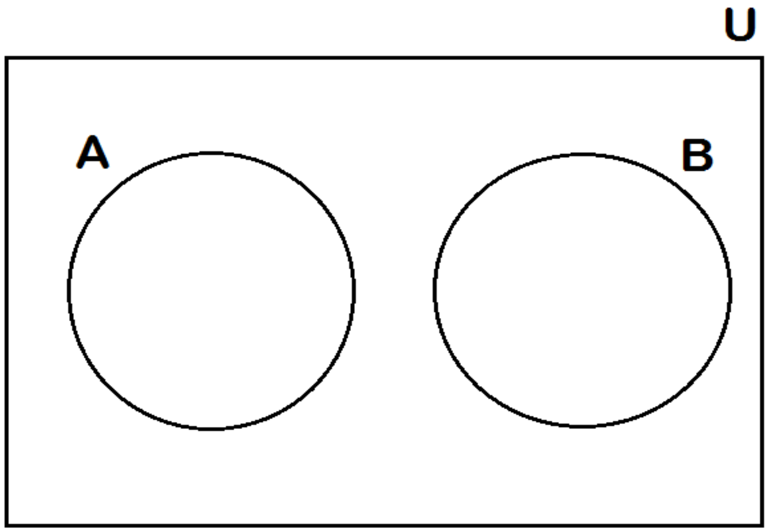
Using the sets above,

$$A \cap B =$$

and as a Venn Diagram



In general, it is possible that two sets do not have any elements in common. For the moment, assume $B = \{7, 8, 9\}$, then $A \cap B = \varnothing$ and as a Venn diagram we have a picture like:



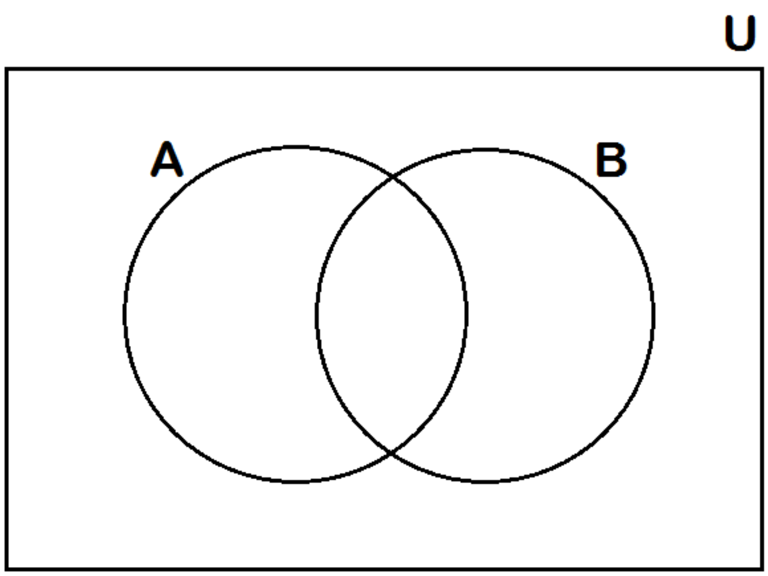
Definition 3 (Complement)**.** *The* complement *of a set A is the new set, denoted A', which consists of all elements which are in U, but not in A.*

$$A' = \{x \in U | x \notin A\}.$$

Using the sets above,

$$A' =$$

and as a Venn Diagram



Example 5. *Let* $A = \{3, 6, 9\}$*,* $B = \{3, 4, 5, 6, 7\}$*,* $C = \{4, 5, 7\}$*, and* $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$*. Find* $A \cup B$*,* $A \cap B$*,* $A \cap C$*, and* B'*.*

Example 6. *Let* $R = \{1, 2, 3, 4\}$*,* $S = \{1, 3, 5, 7\}$*,* $T = \{2, 4\}$*, and* $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$*. Find* $R \cup S$*,* $R \cap S$*,* $S \cap T$*, and* S'*.*

Finally, the last operation we have on sets here is to count the number of elements in a set. We will denote the number of elements in a set A by $n(A)$. In our running example of $A = \{1, 2, 3, 4, 5\}$, $B = \{3, 4, 5, 6, 7\}$, and $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, we have the following facts:

$$\begin{aligned} n(A) &= \\ n(B) &= \\ n(A \cap B) &= \\ n(A \cup B) &= \\ n(A') &= \\ n(A \cap B') &= \\ n(\varnothing) &= \end{aligned}$$

Example 7. *Let the universal set U be the set of positive integers less than or equal to* 100*. Let A be the set of multiples of* 3 *in U, and let B be the set of multiples of* 5 *in U.*

(a) *Find $n(A \cap B)$, $n(A \cap B')$, $n(B \cap A')$, and $n(A' \cap B')$.*

(b) *Draw a Venn diagram with circles labeled A and B, indicating the numbers of elements in the subsets of part (a).*

Solution.

Example 8. *Let the universal set U be the set of positive integers less than or equal to* 100*. Let A be the set of multiples of* 4 *in U, and let B be the set of multiples of* 7 *in U.*

(a) *Find $n(A \cap B)$, $n(A \cap B')$, $n(B \cap A')$, and $n(A' \cap B')$.*

(b) *Draw a Venn diagram with circles labeled A and B, indicating the numbers of elements in the subsets of part (a).*

Section 7.3 - Basic Counting Principles

Addition Principle. Suppose that there are 15 male and 20 female Physics majors at a university. How many total Physics majors are there?

Now, suppose that every freshmen who is majoring in Chemistry is enrolled in Calculus or in History. If there are 20 freshmen Chemistry majors enrolled in Calculus and 15 freshmen Chemistry majors enrolled in History. How many total freshmen Chemistry majors are there?

Theorem 1 (Addition Principle for Counting)**.** *For any two sets A and B,*

Example 9. *According to a survey of business firms in a certain city,* 345 *firms offer their employees group life insurance,* 285 *offer long-term disability insurance, and* 115 *offer group life insurance and long-term disability insurance. How many firms offer their employees group life insurance or long-term disability insurance?*

Multiplication Principle.

Example 10. *Suppose a store has* 3 *types of shirts, and in each type of shirt, they have* 4 *colors available. How many options are available?*

Solution.

Theorem 2 (Multiplication Principle for Counting)**.**

(1) *If two operations O_1 and O_2 are performed in order, with N_1 possible outcomes for the first operation and N_2 possible outcomes for the second operation, then there are*

possible combined outcomes of the first operation followed by the second operation.

(2) *In general, if n operations $O_1, O_2, ..., O_n$ are performed in order, with possible number of number of outcomes $N_1, N_2, ..., N_n$, respectively, then there are*

possible combined outcomes of the operations performed in the given order.

Example 11. *Suppose a 6-sided die and a 12-sided die are rolled. How many different possible outcomes are there?*

Example 12. *Suppose we have a list of 8 letters that we wish to make code words from. How many possible 4-letter code words can be made if:*

(a) *letters can be repeated?*

(b) *no letter can be repeated?*

(c) *adjacent letters cannot be alike?*

Example 13. *Repeat the above example, but with a list of* 10 *letters to choose from and with code words that are* 5 *letters long.*

Example 14. *There are* 30 *teams in the MLB. Suppose a store sells both fitted and snapback baseball caps. Suppose the store carries standard and alternate versions of the fitted cap for each team, but only the standard version of the cap for the snapback cap. How many total different baseball caps do they sell?*